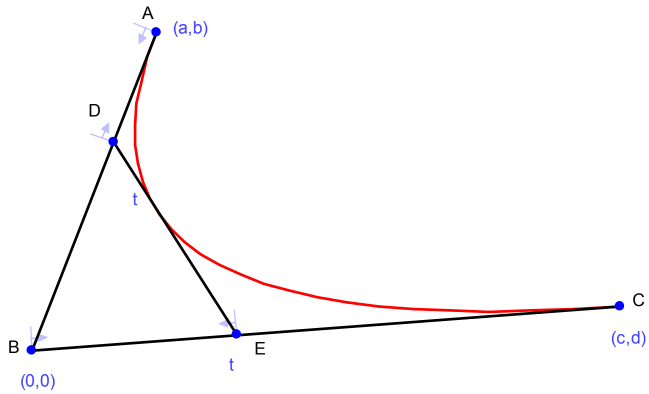


$$\Rightarrow b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d + a^2 \cdot d^2 + Y^2 \cdot \left(a^2 + 2 \cdot a \cdot c + c^2\right) + X \cdot Y \cdot (-2 \cdot a \cdot b - 2 \cdot b \cdot c - 2 \cdot a \cdot d - 2 \cdot c \cdot d) + X^2 \cdot \left(b^2 + 2 \cdot b \cdot d + d^2\right) + Y \cdot \left(2 \cdot a \cdot b \cdot c - 2 \cdot b \cdot c^2 - 2 \cdot a^2 \cdot d + 2 \cdot a \cdot c \cdot d\right) + X \cdot \left(-2 \cdot b^2 \cdot c + 2 \cdot a \cdot b \cdot d + 2 \cdot b \cdot c \cdot d - 2 \cdot a \cdot d^2\right) = 0$$

an ellipse. Which of these three options does it look like?

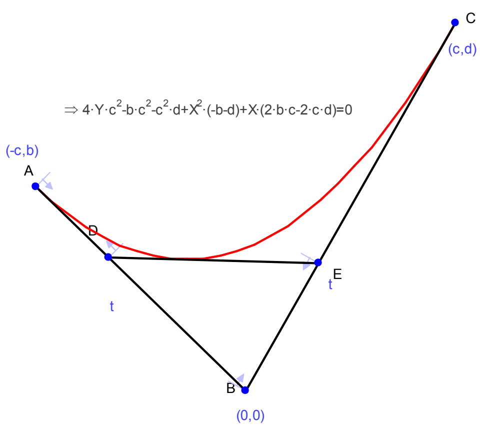
It's not so easy to tell, as the equation is quite complicated. If the curve was a parabola and was lined up so that its axis of symmetry was parallel to the Y axis, we know that it would have the form:

$$Y = aX^2 + bX + c$$

In other words, it would have no Y^2 term, and no XY term.

Look at the coefficient of Y^2 in the above equation. Under what circumstances would it be 0?

The coefficient factors to $(a+c)^2$, which is 0 when $a = -c$.

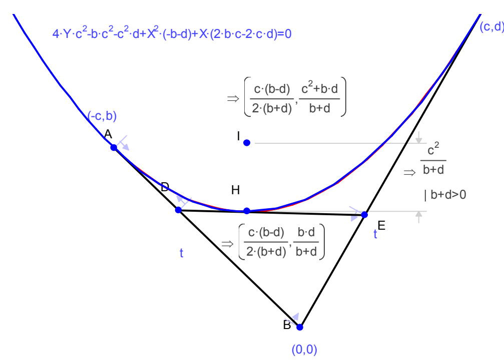


If we make $a = -c$, we see that the equation has lost both the terms in Y^2 and in XY, and hence we know that it is a parabola.

That's all very well, but can we always position the string art so that points A and C are the same distance either side of the Y axis? Indeed we can, we simply rotate about B until the median of the triangle ABC is vertical.

Focus and Vertex

To define a parabola geometrically, we need to know where its focus and vertex are. We can get the coordinates of these points by creating a parabola in Geometry Expressions and giving it the equation of the envelope curve:



If the triangle is equilateral (see below), then

$$b = d = \sqrt{3}c\,.$$

What are the coordinates of the focus?

Can you show that this is the centroid of the triangle, and hence all three parabolas in the picture below have the same focus?