

Trammels

Archimedes Trammel is a device consisting of a handle attached at two pivot points to sliders which can slide to and fro in perpendicular slots (figure 1). In this activity, we will investigate the curve formed by the trammel.

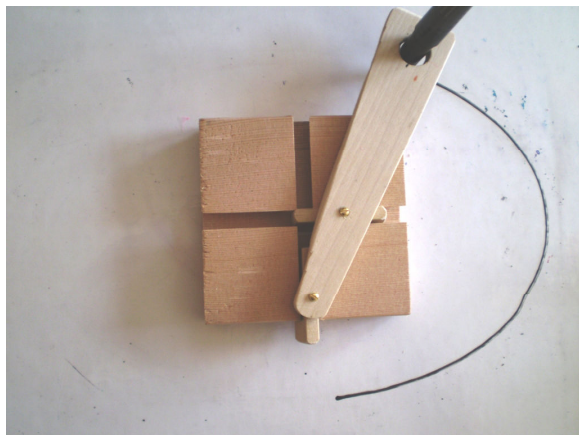


Figure 1: Archimedes trammel

We can model the trammel simply in Geometry expressions by drawing a line segment AB such that A is snapped to the y-axis, and B is free. We then create a point C at the intersection of AB with the x-axis. The model is further constrained by specifying the length AC to be a, and the length AB to be b (figure 2).

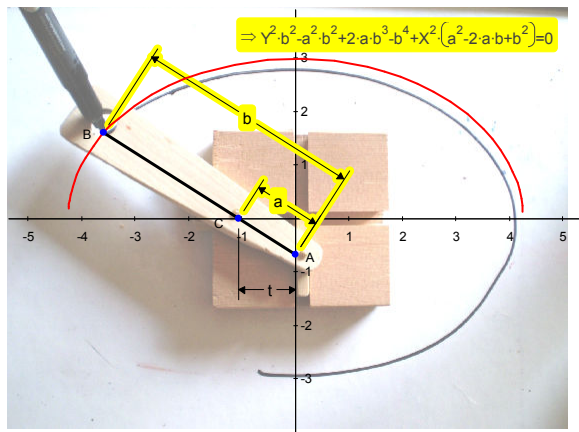


Figure 2: Geometry Expressions model of the trammel computes the equation of the curve and verifies that it is an ellipse

In order to create a curve, we need to specify a parameter of the resulting motion. A suitable parameter is the distance between the point C and the y-axis. In figure 2, we have specified this distance to be t.

Create the curve by selecting point B then choosing the locus tool, and selecting t as the parameter.

Having created the curve, you can select it and ask for its implicit equation.

Examination of the equation shows that it has terms in X^2 , Y^2 and a constant term. It is therefore in the correct form for an ellipse (or hyperbola, but I think it is visually clear that it is an ellipse).

Given that this is an ellipse, can you work out the lengths of its major and minor axes?

The major axis length is clear from figure 2. To compute the minor axis, you could try this starting from the equation, setting $X=0$ and solving for Y , or perhaps you can work it out directly from the geometry by looking at the trammel picture in figure 2. The answer is seen in figure 3.

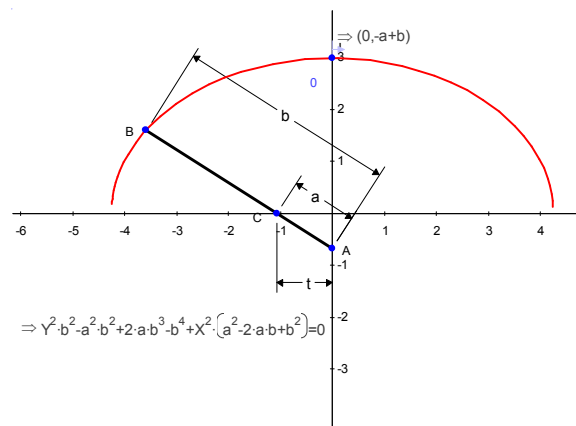


Figure 3: Minor axis of the ellipse (location of point at parametric location 0 on the curve)

Ellipse Foci

An alternative method of drawing an ellipse is to pin a piece of string at two points (called foci), and trace round keeping the string taut. Where would we need to put the foci and what length string would we need to trace out the same ellipse as the trammel?

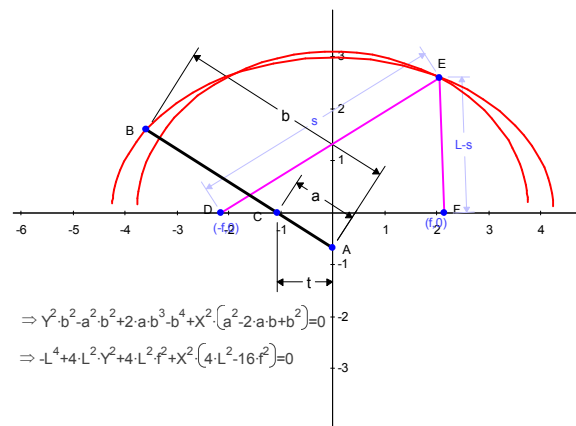


Figure 4: Ellipse formed using a string length L pinned at $(f, 0)$ and $(-f, 0)$