Beyond the Ellipse

Figure 4: String gets caught up on a stick

Keeping going, the string wraps around the stick. What shape is the next portion of the curve? Looking at the picture we can see that the curve is a portion of a circle, at least until we hit the string between the sticks. What is the radius of the circle?

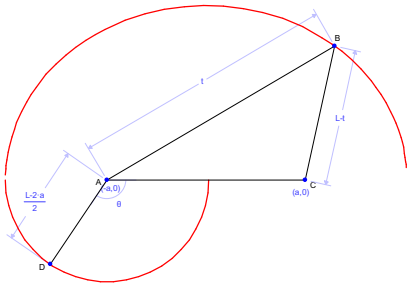

Figure 5: The second part of the curve is a semi circle

When we complete the semi circle, the string catches up with the taut string between the sticks and we trace another ellipse-like curve:

Figure 6: String interference

This can be modeled in Geometry Expressions by creating a new triangle and specifying that the distance from the first stick is s, and the distance from the second stick is L-3s.

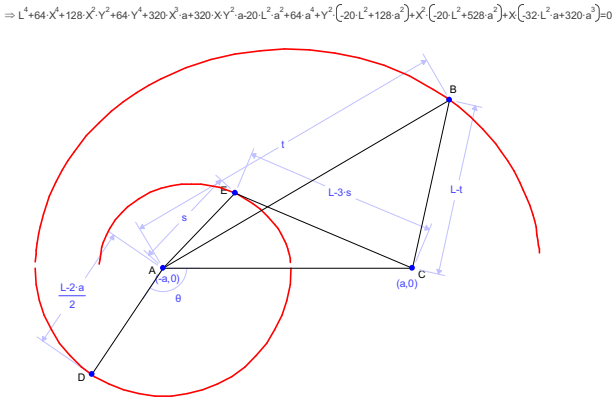

Figure 7: Modeling the next 180 degrees of the curve

Is this curve actually part of an ellipse? What order is the curve?

Figure 8: The first 540 degrees of the curve

If we keep going with this process (difficult in real life, but easy on a computer), we can see that we will get a sequence of alternating semi-circles and other curves. What are the radii of the semi circles?

We can create the generic case of the upper curve by setting the distances in the above figure to s and L-n*s.

The expression is complicated, but we can see that in general it is a 4^{th} order curve (except of course when n=1 or -1, in which case all but the 2^{nd} order terms vanish).