We are interested in the location of the central cusp. First in the situation where a = 1. Change the coordinates of C to (0,-1), and then display the parametric equation of the caustic curve. (Fig 4)

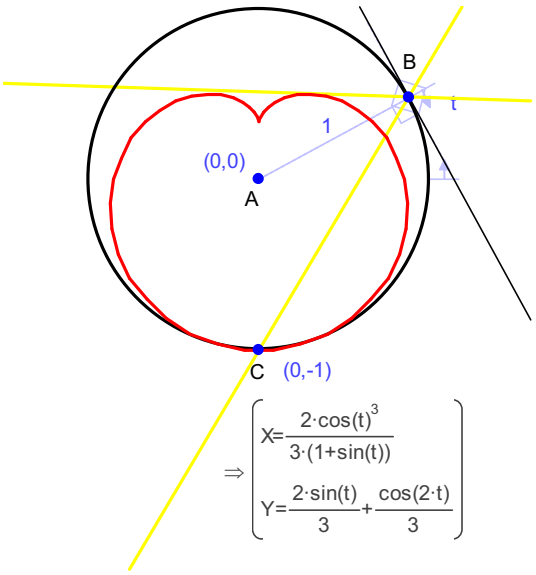

Figure 4: Parametric equation of the caustic with light source on the circumference

What value will t have when the reflected ray passes through the cusp (remember t is the angle in radians of AB)?

What is the y value of the curve for that value of t?

Does that result match your observations?

Cusp on the Circumference

As the light source moves towards the center of the cylinder, the caustic curve is no longer fully contained in the cylinder. However, at some point, it re-enters to give the shape in Figure 5. Again, using the circular graph paper, estimate the location of the central cusp at this point.

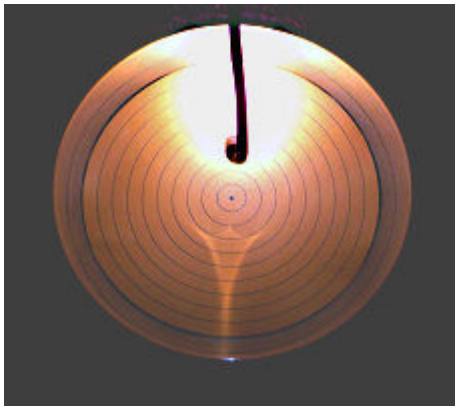

Figure 5: The caustic when the far cusp touches the circumference.

In Geometry Expressions, change the coordinates of C back to (0,-a) and drag C towards the center of the circle until the second central cusp appears:

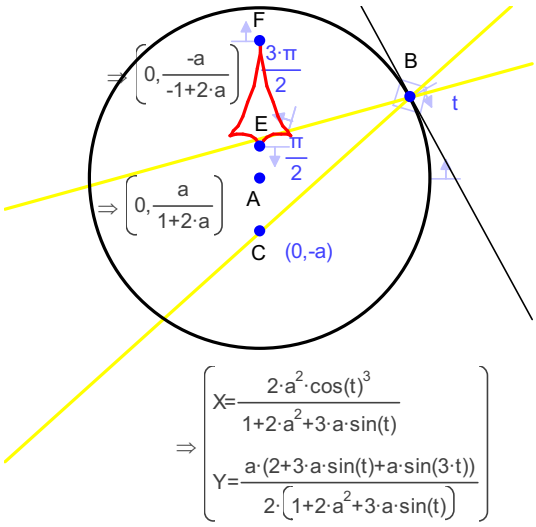

Figure 6: Curve equations for general a. Points E and F are put on the curve and constrained to be at parametric locations π/2 and 3π/2

In figure 6, we have put points E and F on the curve and constrained them to be parametric distance π/2 and 3π/2 along the curve. We have displayed the coordinates of E and F

What value of **a** will put F at the circumference?

In which case, where will E lie?

Does this match your observations?

Further Questions

As **a** gets larger and larger ("tends to infinity"), where does the cusp at E go?

Can you verify this experimentally? (The room lighting will be effectively at infinite distance, unless you happen to have placed your tube directly under it.)

For a curve (X(t),Y(t)), cusps occur where the derivatives of X with respect to t and Y with respect to t simultaneously vanish.

If you have access to a computer algebra system, you might like to try to determine the parameter value (t) of the other caustics.

Can you interpret this geometrically?

Can you derive the coordinates of the cusps?

Can you display the locus of the cusps as a varies?