

A Geometry Expressions Activity

Figure 4 shows a general arrangement where the foci are at point $(f,0)$ and $(-f,0)$, and the string is length L .

You can try dragging the foci and the point E to match the trammel curve. As there are two variables, f and L , this task is not easy. If we could fix one of the parameters, then drag the other the problem would be simpler.

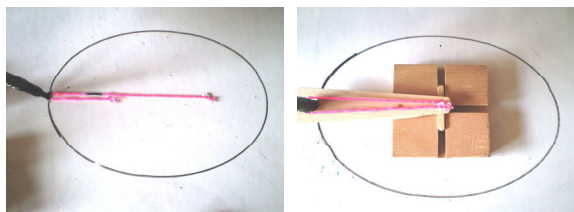


Figure 5: As the two foci are equidistant from the center of the ellipse, the string length should be twice the length of the trammel handle

If we imagine aligning the string with the ellipse's major axis (figure 5), we can observe that the string length must be twice the length of the handle. (Imagine pulling the foci gradually towards the center, while keeping the string taut with the pen). Figure 6 shows the result of setting the string length to be $2b$.

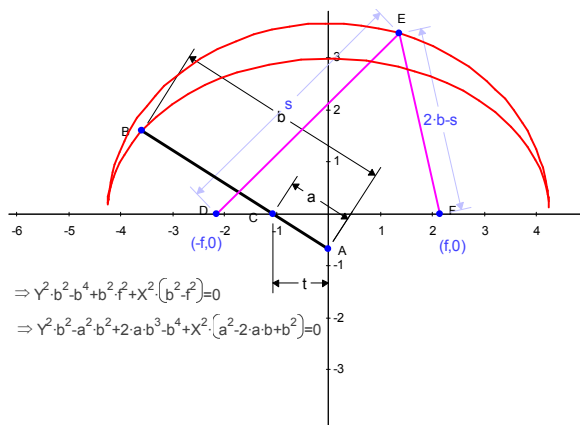


Figure 6: Setting the string length to $2b$

It is now easy to drag a focus to line up the curves, but can we work out an algebraic expression for f in terms of a and b ?

We could examine the equations of the curves. The coefficients of Y^2 are equal, what value of f would make the coefficients of X^2 equal?

Alternatively we could think geometrically. Looking at figure 7, can you imagine a right angled triangle which is formed when the pen passes through the minor axis of the ellipse? We already know the length of the vertical side of this triangle (fig. 3). What is its hypotenuse? What is the length of its horizontal side?

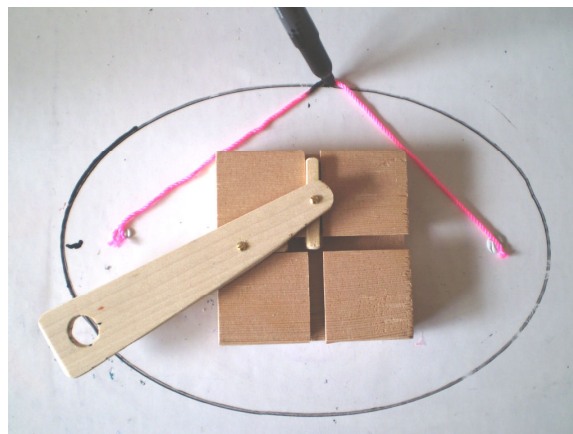


Figure 7: The string stretched approximately through the minor axis of the ellipse

Either algebraically or geometrically, we should

derive the solution $f = \pm \sqrt{2ab - a^2}$. Figure 8 shows the result of using these foci, and we observe that the curves align geometrically and have the same algebraic equation.

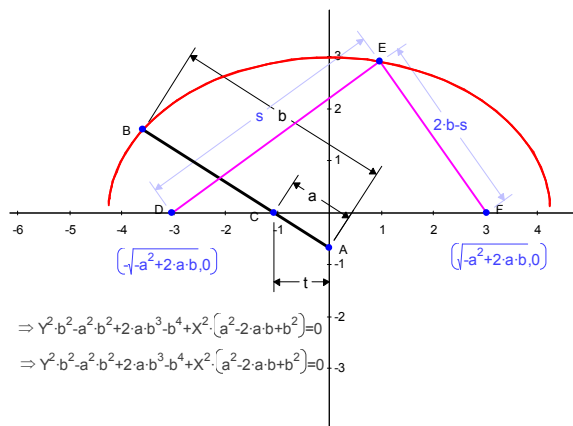


Figure 8: Foci at $x = \pm \sqrt{2ab - a^2}$ aligns the two curves

For further thought

Figure 9 shows a trammel with 3 slots. Why is it surprising that this moves? What curve does the handle trace out? Why?



Figure 9: A trammel with 3 slots.