String Art & Envelopes

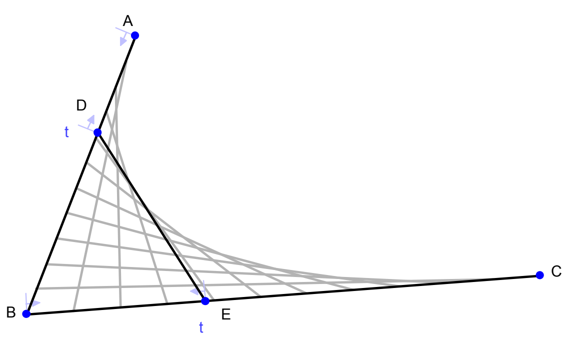
One of the interesting things about string art is that you can generate curves out of straight lines. Or at least it seems like you can.

In this activity we'll investigate the mathematics of these string art curves. Here's an example:

We're interested in the cut of this boat's jib, and in particular its curved edge.

The string art design starts with two lines along the straight sides of the sail. (Notice these lines are not the same length). 12 evenly spaced nails are hammered into each line. Corresponding pairs of nails are joined by string.

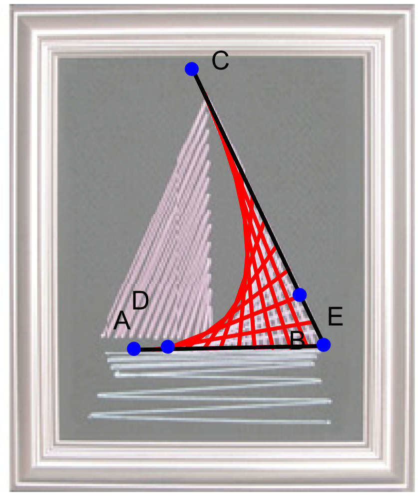
This can be modeled in Geometry Expressions using the point proportional along a line constraint:



In the diagram, D is proportion ***t*** along the line AB, while E is proportion ***t*** along BC.

DE represents a generic piece of string, and the Geometry Expressions Trace command lets you see the whole family.

You can insert the string art into Geometry Expressions and drag the points A,B and C to fit the model to the sail (it is convenient to lock the parameter ***t*** first).

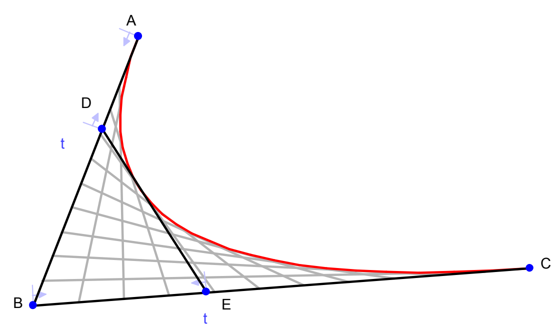


We have visualized the curve with a family of straight lines in Geometry Expressions. The really nice thing about the software, is that we can create the actual curve. To do that we just select the line DE in our model, and request its locus.

Envelopes

When you ask for the locus of a line in Geometry Expressions, what you get is a curve called the "envelope". For each value of the parameter t, a line is defined. The envelope is the curve which is tangential to each of these lines.

Here it is for our string art figure:



What kind of curve is this?

Geometry Expressions can give us its equation. First, we'll set the coordinates of A,B,C. We'll put B at the origin, A at coordinates (a,b) and B at coordinates (c,d).

The equation for the curve may look a bit intimidating, but remember a,b,c and d are just parameters, and can be ignored at first, as we focus on X and Y. Close examination shows that we have terms in X, Y, X^2, Y^2, and XY. We can infer that our curve is a conic section – a hyperbola, a parabola or