Light Caustics

If you look at light reflected within a cylinder, you see a bright curve caused by the light being concentrated into a specific region. This curve is called a caustic.

You can see this curve readily if you put a shiny wedding ring on a piece of paper. But all sorts of cylindrical objects will generate caustics, including coffee mugs, pots and pans, and in the pictures below, a piece of perspex tubing.

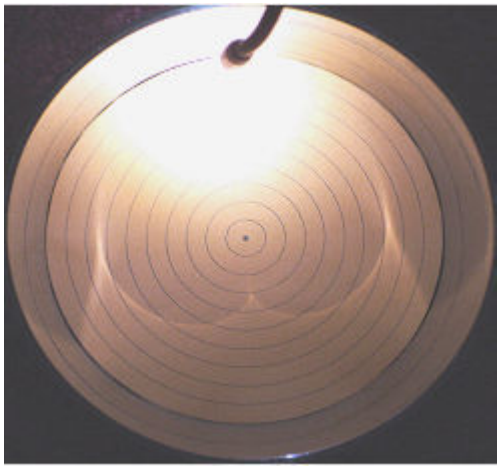

Figure 1: Caustic inside a cylinder with the light part of the way from the circumference to the center.

When the light source is outside the cylinder, as we will see, the caustic rapidly becomes effectively equivalent to one generated from an infinitely distant light. Here we want to look at the shapes generated by a light source inside the cylinder. For this we need to find a big enough cylinder and a small enough light. With this kind of apparatus, we can generate an interesting family of caustic curves:

In order to make quantitative observations on the curve, you need to create some circular graph paper. You can do this in Geometry Expressions itself. Draw 10 concentric circles, then set the radius of the first circle to be r, the radius of the second to be 2*r, the third to be 3*r, etc. Print it out and see how much bigger or smaller than the cylinder your 10^{th} circle is. Then adjust r accordingly and print out again.

Light on the circumference

Figure 2 shows the caustic when the light source is placed at the circumference of the cylinder. Using the circular graph paper, estimate how far from the center the cusp is in this situation.

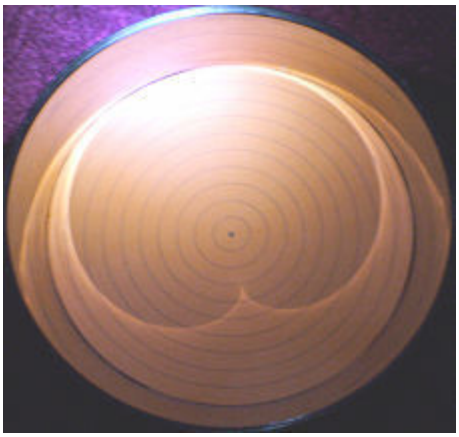

Figure 2: Caustic curve with the light at the radius of the cylinder

We can model the behavior of a single beam of light in Geometry Expressions. First we create a circle AB, and constrain its center to be (0,0) and its radius to be 1. We then constrain the parametric location of B on the curve to be t. This has the effect of specifying the AB to be angle t (in radians) from the x axis. Geometry Expressions allows you to reflect in a line, but not in a curve. However reflection of light in a cylinder is equivalent to reflection in the tangent to the cylinder. Hence we draw a line through B and constrain it to be tangent to the circle. We now create point C, constrain its location to be (0,-a), and draw an infinite line through C and B. We complete the model by reflecting BC in the tangent line.

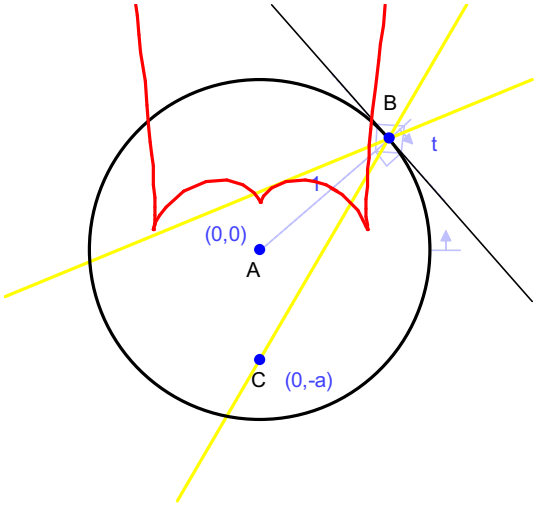

Figure 3: Geometry Expressions model of a beam of light reflected in a cylinder.

To display the caustic curve, you need to create the envelope of the reflected line. This is done by selecting the reflected line and using the Locus tool. With the caustic curve displayed, try dragging C closer or further away from the center of the circle. Do the curve shapes match those you observed?